A View of P Systems from Information Theory

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Abstract. In this work we propose new view of P systems by using the framework of Information Theory. Given a cell-like P system with communication and evolution rules, we analyze the amount of information that it holds as the result of symbol movements across the membranes. Under this approach, we propose new definitions and results related to the information of P systems and their entropy. In addition, we propose a new working manner for P systems based only in the entropy evolution during the computation time.

Keywords: Communication P systems · Information theory · Entropy

1 Introduction

P systems were introduced as a computational model inspired by the information and biochemical product processing of living cells through the use of membrane communication. In most of the works about P systems, information is represented as multisets of symbols/objects which can interact and evolve according to predefined rules. From the beginning, the most important component of the system has been the kind of rules that it holds. There have been different proposals to define the rules of the system such as evolution rules, communication rules, active rules to create/dissolve membrane structures, active rules with polarization, and so on and so forth [13].

Here, we pay attention to the following fact: the rules of a P system produce/consume new symbols in different regions of the system. So, they can be considered information regulators that act over a region, which can be considered information senders and receivers in a pure communication system. Hence, the behavior of the P system can be analyzed from the Information Theory point of view. In this context, the main concept to be defined and applied is the concept of *entropy*. From the definition of entropy, we can analyze any P system through the characterization of the information at every region according to its membrane structure and rules.

Furthermore, if we consider the P system as a metaphor of a living system, then, by applying thermodynamic laws, the system will tend to increase

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its entropy. Hence, we can establish a new working manner of any P system that selects the rules and its application number as a function of incrementing the entropy. This new way of working is named *entropic manner* and will be explained in more detail.

The structure of this work is as follows: First, we give the basic definitions needed in this work and related to Information Theory, multisets and P systems. Then, we establish different entropy definitions for the structure and ingredients of some P systems. We overview some algorithmic ways to calculate the entropies of the system, and we consider probabilistic/stochastic and non-probabilistic/non-stochastic systems. In the next section, we define P systems working in an entropic way and we overview the application of rules under this approach. Finally, we propose some additional research topics related to this approach and we report work in progress.

2 Basic Concepts

We will introduce basic concepts related to multisets, Information Theory and P systems. We suggest to the reader the references [12,13] to introduce membrane computing, and the books [5,15] to introduce Information Theory. We will provide some definitions from multiset theory as exposed in [17].

Information Theory. We can suggest to the reader the books [5,15] and the classical work by C.E. Shannon [16] in order to have a full view of Information Theory.

An *information source* is defined by the tuple (S, P), where S is an alphabet (random variables) and P is a probability distribution over S. A cornerstone of Information Theory is the concept of *entropy* which is attached to information sources. The entropy of an information source I, with an alphabet S and probability distribution $P : S \to [0, 1]$ is defined as

$$H(I) = -\sum_{a \in S} P(a) \cdot \log_2 P(a)$$

Observe that we are working with trivial codes where the alphabet of an information source is its encoding. We have fixed the base 2 for the logarithmic functions, so the information entropy is described in bits. The change from a binary base to a different one can be easily carried out in a logarithm base change. In addition, we can consider the conditional and joint entropies of two random variables X and Y, respectively $H(X \mid Y)$ and $H(X, Y)$, by using the appropriate probability distributions.

Given, two probability distributions p and q over S, the *relative entropy* or *Kullback-Leibler distance* is defined by

$$D(p \mid\mid q) = \sum_{a \in S} p(a) \cdot \log_2 \frac{p(a)}{q(a)}$$

and, for two random variables X and Y, with a joint probability distribution $p(x, y)$, the *mutual information* $I(X, Y)$ is defined as

$$I(X,Y) = \sum_{x \in X} \sum_{y \in Y} p(x,y) \cdot \log_2 \frac{p(x,y)}{p(x) \cdot p(y)}$$

Observe that the mutual information is the relative entropy between the joint distribution and the product distribution. The following relations between mutual information and entropies hold

- $I(X,Y) = H(X) + H(Y) - H(X,Y)$,
- $I(X,Y) = H(X) - H(X \mid Y) = H(Y) - H(Y \mid X)$.

Multisets and P Systems. Let D be a set. A multiset over D is a pair $\langle D, f \rangle$ where $f : D \longrightarrow \mathbb{N}$ is a function. If $A = \langle D, f \rangle$ and $B = \langle D, g \rangle$ are two multisets then the removal of multiset B from A, denoted by $A \ominus B$, is the multiset $C = \langle D, h \rangle$ where, for all $a \in D$, $h(a) = max(f(a) - g(a), 0)$, and their sum, denoted by $A \oplus B$, is the multiset $C = \langle D, h \rangle$, where for all $a \in D$, $h(a) = f(a) + g(a)$. We will say that $A = B$ if the multiset $(A \ominus B) \oplus (B \ominus A)$ is empty that is $\forall a \in D$, $f(a) = 0$.

The size of any multiset M is the number of elements that it contains and will be denoted by $|M|$. In the following, we will represent multisets by using strings over the alphabet induced by D. Hence, for every alphabet $D = \{a_1, a_2, \cdots, a_n\}$ and for every string $x \in D^*$, we will use the well known Parikh vector defined by $\Psi_D(x) = (|x|_{a_1}, |x|_{a_2}, \ldots, |x|_{a_n})$ where $|x|_{a_i}$ is the number of occurrences of the symbol a_i in x. Observe that, in this case, the length of the string is the size of the multiset that it defines. Finally, for any multiset denoted by the string x, $alph(x)$ denotes the set D that defines the multiset x.

A *cell-like P* system of degree m with *communication* rules is a construct

$$\Pi = (V, \mu, w_1, \cdots, w_m, R_1, \cdots, R_m, i_0),$$

where:

- V is an alphabet (the *objects*)
- μ is a membrane structure consisting of m membranes
- w_i, $1 \leq i \leq m$, is a string representing a multiset over V associated to the region i
- R_i, $1 \leq i \leq m$, is a finite set of rules of the form (u, v) with $u \neq \lambda$ and $v \neq \lambda$ (evolution rules), $(u, out; v, in)$ with $u \neq \lambda$ and $v \neq \lambda$ (antiport rule) and (x, out) or (x, in) with $x \neq \lambda$ (symport rule). The strings u, v and x are defined over the alphabet V, and λ denotes the empty string.
- i_0 is a number between 1 and m and it specifies the *output* membrane of Π (in the case that it equals to ∞ the output is read outside the system).

Observe that, in the previous definition, we have omitted an output alphabet, a catalyst alphabet and dissolution rules. In addition, we have omitted priorities

in the rule sets and other communication rules with explicit address. The main reason is that we want to establish a preliminary analysis with the most simple P systems. We have relaxed the definition of P systems by using standard symbol-object ingredients together with antiport (symport) rules. Furthermore, we have not fixed a working manner of the system. The main reason is that all the definitions and results that we propose in this work are valid for any of the working modes proposed in the literature [13].

Given a P system $\Pi = (V, \mu, w_1, \cdots, w_m, (R_1, \rho_1), \cdots, (R_m, \rho_m), i_0)$, a *configuration* of Π at time t will be denoted by $(\mu, w_1^t, w_2^t, \cdots w_m^t)$ where w_i^t is the multiset of objects that region i holds at time t. Obviously, for every region i $w_i^0 = w_i$ (the initial configuration). A *computation* is defined as a (finite) sequence of configurations $C_0, C_1, \cdots, C_p$ where every configuration follows from the previous one by applying the rules over the multisets in a predefined (maximal, minimal, sequential, etc.) manner. Observe that, given that the system has no creation nor dissolution rules, it is no necessary to include μ in the configuration.

Given that a P system is a non-deterministic computational device, it is quite useful the use of computation trees instead of computation sequences. A computation tree is defined by a set of nodes (configurations) with the following conditions: First, the root is the initial configuration of the system and every son of an internal node is defined by the application of rules over the multiset of every region. Given that the system is non-deterministic then every son of an internal node is defined by one possible combination of rule applications over the configuration that it defines. Figure 1 shows a graphical view of this definition.

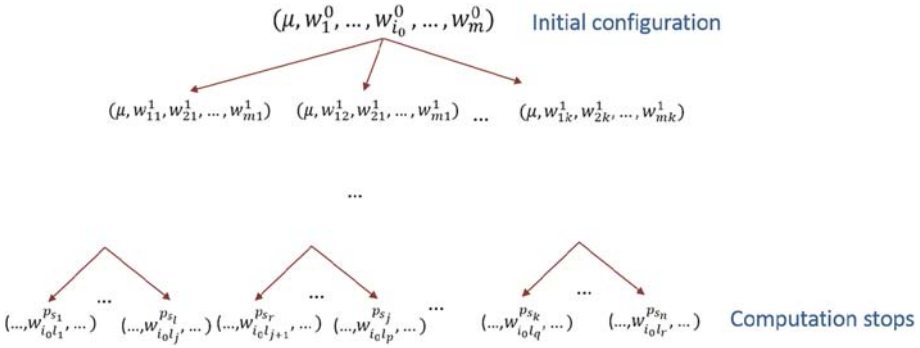

Fig. 1. A computation tree of a P system

An important concept to define entropies for P system is to establish whether the objects are produced in a stochastic/probabilistic manner or not. For the former, the probabilistic distribution is carried out by the definition of stochastic/probabilistic P systems such as Dynamical Probabilistic P systems (DPP) [14] or Population Dynamics P Systems (PDP) [4], together with their simulation algorithms [2,10,11].

3 The Entropy of a P System

We will define the entropy of a P system by analyzing how the multisets at every region evolve according to the rules of the system. First, we define the entropy of the multisets of the regions and, then, the entropy of a P system. In order to carry out a rigorous analysis, we need to distinguish whether the P system is a probabilistic/stochastic one or not. In the following, we discuss both cases.

The Non-probabilistic/Non-stochastic Case. We consider that every multiset at every region in the P system is defined by the application of rules in a pure non-deterministic non-probabilistic/non-stochastic case. Hence, the multisets at every region during the computation reflect, in an isolated way, the information production that can be considered to calculate the entropy. We will introduce a definition of entropy that is related to every multiset without any probabilistic information source.

Definition 1 (self-referred entropy of a multiset). *Let us consider a multiset $A = \langle D, f\rangle$ represented by x. The self-referred entropy of x is defined as*

$$H_s(x) = -\sum_{a\in D} fr(a)\cdot log_2 fr(a)$$

where $fr(a) = \frac{|x|_a}{|x|}$.

Observe that, in the previous definition, the probability distribution has been substituted by the frequency of appearance of every object at the region ($fr(a)$).

In the following, we analyze the evolution of self-referred entropies according to the computations of the system.

Definition 2. *Let Π be a P system of degree m and $c_t = (\mu, w_1^t, \cdots, w_m^t)$ be a configuration of the system during a computation at time t. Then*

1. *The* absolute entropy *of Π at time t is $H^t_{abs}(\Pi) = \sum_{1\leq i\leq m} H_s(w_i^t)$*
2. *The* maximal entropy *of Π at time t is $H^t_{max}(\Pi) = max\{H_s(w_1^t), \cdots, H_s(w_m^t)\}$*
3. *The* minimal entropy *of Π at time t is $H^t_{min}(\Pi) = min\{H_s(w_1^t), \cdots, H_s(w_m^t)\}$*
4. *The* average entropy *of Π at time t is $H^t_{avg}(\Pi) = \frac{H^t_{abs}(\Pi)}{m}$*
5. *The* holistic entropy *of Π at time t is $H^t_{hol}(\Pi) = H_s(w_1^t w_2^t \cdots w_m^t)$*

Property 1. The following relation holds

$$H^t_{min}(\Pi) \leq H^t_{avg}(\Pi) \leq H^t_{max}(\Pi) \leq H^t_{abs}(\Pi) \leq H^t_{hol}(\Pi)$$

Proof. Trivial from the definitions.

The question about the computation of the entropy of a P system is completely based on the calculation of the different multisets at every region, according to the rules that affect to that region. Hence, at time t the multiset w_i^t will evolve, in the next transition, to the multiset $w_i^t \ominus left(R,i) \oplus right(R,i)$, where $left(R,i)$ is a multiset based on the left hand side of the rules that affect to the region i, and $right(R,i)$ is a multiset based on the right hand side of the rules that affect to the region i. Hence, the way to calculate the different entropies defined before is carried out by the following execution scheme:

1. Calculate the following configuration of the system according to its working manner (minimal, maximal, sequential, etc.)
2. For every multiset w_i^{t+1} calculate $H_s(w_i^{t+1})$.
3. Calculate the different entropies based on the self-referred entropies of the system.

The Probabilistic/Stochastic Case. In this case, we assume that every rule in the system has a *kinetic (stochastic)* real value that influences the application of the rules and the way to obtain the new configurations during the computation time. The main systems that have been proposed to calculate the configurations in a probabilistic/stochastic manner have been Dynamical Probabilistic P systems (DPP) [14], including the τ-DPP systems [3], or Population Dynamics P Systems (PDP) [4]. In both cases, there are simulation algorithms that manage the configurations evolution in order to produce the desired stochastic/probabilistic effect [1,2,8–11].

In this case, the definition of entropy should be based on the appearance probabilities of every symbol at every region. Here, the symbol probabilities come from different probabilities sources according to rules at (different) regions that produce the same symbol with different probabilities.

For example, let us consider the P system of Fig. 2. There are three regions and the object b can be produced at region 1 by using the rule r_1 at region 2 or the rule r_2 at region 3. While the rule r_1 is applied at every computation step (it has probability 1.0 to be applied), the rule r_2 should compete with rule r_3 to be applied (if we suppose a uniform distribution of kinetic constants). So, the object 'b' at region 1 has probability 1.0 or probability 0.5 depending on the rule that has produced it.

We evaluate every symbol at every region by establishing where it was created in order to reflect this situation. Then, given a P system Π, let us suppose that, at region i, the object 'a' can be produced by the set of rules R_a^i, with cardinality $|R_a^i|$. This set is easily deduced by the set of rules at every region adjacent to region i. We propose a naive approximation to the probability of symbol 'a' at region i, and computation time t, as follows

$$P_i^t(a) = \frac{\sum_{r \in R_a^i} Pr^{t-1}(r)}{|R_a^i|}$$

where $Pr^{t-1}(r)$ is the probability of applying the rule r at time $t-1$ that is calculated in a stochastic/probabilistic way as referred above.

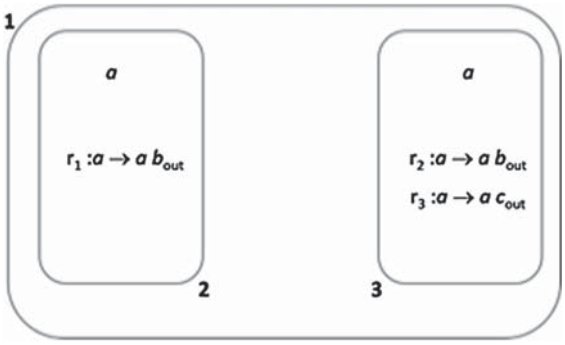

Fig. 2. A P system with competing rules

Now, we can reformulate the entropy of any multiset that reflects the stochastic/approximation approach.

Definition 3 (entropy of a multiset at time t). *Let Π be a P system and let x be a multiset at time t located at region i. The entropy of x at time t is defined as*

$$H^t(x) = -\sum_{a \in alph(x)} |x|_a \cdot P_i^t(a) \cdot log_2 P_i^t(a).$$

If we substitute at Definition 2, the entropy $H_s(w_i^t)$ by $H^t(w_i^t)$ then we have the corresponding entropies of the P system into a stochastic/probabilistic approach. Additionally, we can calculate the entropy of the system by executing the following scheme:

1. Calculate the following configuration of the system and the probabilities of the rules according to a pre-defined algorithm [1,2,8–11].
2. For every multiset w_i^{t+1} calculate $H^{t+1}(w_i^{t+1})$.
3. Calculate the different entropies of the system based on the entropies at time $t+1$.

Trivially, the relation of Property 1 holds for the definition of entropies under the probabilistic/stochastic approach.

Relative Entropy and Mutual Information Between Adjacent Regions. We will use the mutual information in order to analyze how every multiset at a given region influences the entropy variation for the multisets at its adjacent regions. Let Π be a P system and R_i and R_j two adjacent regions according to its membrane structure, then we propose the following relatives entropies of the random variables W_i^t and W_j^t that reflects the contents of regions R_i and R_j at time t:

1. The mutual information $I(W_i^t, W_j^t)$ reflects the information that is only explained by observing the two regions simultaneously at every computation step

2. The relative entropy $D(P(W_i^t|W_j^{t-1}), P(W_i^t) \cdot P(W_j^{t-1}))$ that measures the causality information effects of one region over the other with respect to their behavior as independent regions.

Observe, that we can generalize the random variables in order to consider the set of regions adjacent to a given one. Hence, we can define the random variable $W^t_{i_1,i_2,\cdots,i_p}$ that takes into account the contents of the regions $R_{i_1}, R_{i_2}, \cdots, R_{i_p}$ which are adjacent to the region R_i at computation time t.

In addition, we can use the computation time to study the information effects of the contents of a given region R_i, at time t, over a non-adjacent region R_k in computation time $t + p$ with $p > 1$.

The scheme to calculate the previous information metrics is based again on the algorithms to calculate the multisets at every region and the estimation of conditional and joint probability distribution.

4 P Systems with Entropic Transitions

The transition modes of a P system has been classically one of the hot topics in membrane computing [7]. In this section we are going to propose a new working manner based on the informational aspects of the system. Furthermore, we can redefine the *confluence* of P systems computation based on this working manner. The following definition introduces this aspect.

Definition 4 (entropic confluence). *Let* $\Pi = (V, \mu, w_1, \cdots, w_m, (R_1, \rho_1), \cdots, (R_m, \rho_m), i_0)$ *be a P system of degree* m. *We will say that* Π *is*

1. *entropic confluent if, for every pair of halting configurations* $(\mu, w_1^t, \cdots, w_{i_0}^t, \cdots, w_m^t)$ *and* $(\mu, w_1^q, \cdots, w_{i_0}^q, \cdots, w_m^q)$, $H_s(w_{i_0}^t) = H_s(w_{i_0}^q)$
2. *complete entropic confluent if, for every pair of halting configurations* $(\mu, w_1^t, \cdots, w_m^t)$ *and* $(\mu, w_1^q, \cdots, w_m^q)$, $\forall i$, $1 \leq i \leq m$, $H_s(w_i^t) = H_s(w_i^q)$
3. *absolute entropic confluent if, for every pair of halting configurations* $(\mu, w_1^t, \cdots, w_m^t)$ *and* $(\mu, w_1^q, \cdots, w_m^q)$, $\sum_{1 \leq i \leq m} H_s(w_i^t) = \sum_{1 \leq i \leq m} H_s(w_i^q)$
4. *maximal entropic confluent if, for every pair of halting configurations* $(\mu, w_1^t, \cdots, w_m^t)$ *and* $(\mu, w_1^q, \cdots, w_m^q)$, $max\{H_s(w_1^t), \cdots, H_s(w_m^t)\} = max\{H_s(w_1^q), \cdots, H_s(w_m^q)\}$
5. *minimal entropic confluent if, for every pair of halting configurations* $(\mu, w_1^t, \cdots, w_m^t)$ *and* $(\mu, w_1^q, \cdots, w_m^q)$, $min\{H_s(w_1^t), \cdots, H_s(w_m^t)\} = min\{H_s(w_1^q), \cdots, H_s(w_m^q)\}$
6. *average entropic confluent if, for every pair of halting configurations* $(\mu, w_1^t, \cdots, w_m^t)$ *and* $(\mu, w_1^q, \cdots, w_m^q)$, $\frac{\sum_{1 \leq i \leq m} H_s(w_i^t)}{m} = \frac{\sum_{1 \leq i \leq m} H_s(w_i^q)}{m}$
7. *holistic entropic confluent if, for every pair of halting configurations* $(\mu, w_1^t, \cdots, w_m^t)$ *and* $(\mu, w_1^q, \cdots, w_m^q)$, $H_s(w_1^t \cdots w_m^t) = H_s(w_1^q \cdots w_m^q)$

Observe that some definitions of entropic confluence implies some other ones. For example, if the system is complete entropic confluent then it is entropic confluent and absolute entropic confluent. An absolute entropic confluent system is average entropic confluent, and so on a so forth.

Entropic Behavior. Inspired by thermodynamics, and as a consequence of the second law of thermodynamics, the *Principle of Maximum Entropy* arises: "*Closed systems evolve to an equilibrium state characterized by a maximum of entropy*". Hence, we can consider a different working manner for P systems based on the growth of the entropy.

Given a P system, we say that the system applies a (maximal, absolute, minimal, average, holistic) *entropic transition* if it applies only the set of rules that increases the (defined) entropies of the system.

Observe that the set of rules that must be applied at every computation step is based in the multisets of objects that every region contains and it can be different at every computation step. In order to illustrate this fact, let us see the following example: Let $r_1 : a \rightarrow aa$ and $r_2 : ab \rightarrow aabb$ be two rules of the same region in a P system. If we consider the multiset $aabb$, then the entropic transition will be carried out by applying rule r_2 instead of r_1, here we obtain the new multiset $aaaabbbb$ which has a maximum self-referred entropy (observe that this maximum is achieved with a uniform distribution of objects). If the multiset is abb then the rule to be applied in order to increase the self-referred entropy is r_1 instead of r_2 and it produces the new multiset $aabb$ (again, we obtain an uniform distribution of objects).

Definition 5. *A P system works in a (maximal, absolute, minimal, average, holistic) entropic manner if at every computation step it only applies (maximal, absolute, minimal, average, holistic)* entropic transitions.

Observe that the simulation of a P system working in any entropic manner, can be a difficult task to be simulated, given that the search of the set of rules needed at every computation step is not a trivial task. In some cases, it can be considered a Multiobjective Optimization Problem with conflicts, given that different functions must be maximized (the entropies functionals at every region), and the increase of entropy at one region could decrease the entropy at an adjacent region (by using symport or antiport rules). Here, some entropy optimization problems can be proved to be $\mathcal{NP}$-optimization ones [6].

Another aspect that must be explored with respect to P systems working in an entropic manner, is the halting criterion. Observe that in our proposal, if no rule increments the entropy of the system, then no rule can be applied at a given time and, accordingly to the usual criterion, the system halts. Obviously, under this point of view the sets of natural numbers and languages that P systems recognize, accept or generate should be defined within this new approach.

5 Conclusion, Further Research and Future Works

In this work we have proposed the Information Theory as a framework to study P systems under a new view. We think that this approach opens new and exciting topics that should be studied in the near time. Among other questions we can point out, the following ones:

1. How does the operational mode (i.e. maximal or minimal parallelism, sequential, etc.) affect to entropy?
2. What is the relationship between confluence and entropic confluence?
3. What is the definition of the entropy of a P system, if the external output is defined?

With respect to the last question, if we consider any P system with a network structure (tissue-like P systems, Spiking Neural P systems, ...) then the entropy should be calculated by taking the output sequence as a pure communication channel. Here, the output alphabet, the time between outputs (specially, in the case of SN P systems) and the definition of randomness are essential aspects that should be considered in order to explore new aspects of this approach.

All these aspects will be studied in the near future and some of them are actually work in progress.

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