A cell-like representation for Markov sources (extended abstract)

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Abstract. In this work, we propose a cell-like P system for the definition of Markov information sources of order m by using stochastic evolution and communication rules that allow the definition of the transition of states of the source in a steady state.

Keywords: Markov sources, cell-like P systems, stochastic P systems

1 Introduction.

Markov sources are classical models to define stochastic processes from the point of view of information theory where the observation of the process defines a Markov chain [4, 12]. Markov sources and chains have been widely used in the computer science area. They are one of the cornerstones for the analysis of communication channels in networks of interconnected elements. Also, they have been used as information sources in simulation systems, and the Hidden Markov Models (HMM) defined by Markov sources have been widely used in pattern recognition and machine learning in different applications.

In this work, we propose the modeling of Markov sources (and Markov chains) as cell-like P systems. Hence, we can use the proposed model as an information stochastic source that can be integrated in a natural way to be used by other P systems. It is not the first time that we relate basic concepts of information theory with P systems: in [11], we proposed different entropy measures to work with P systems. There have been other works that relate P systems with Markov chains. So, for example, in [3], the authors propose a P system constructed in a *semi-uniform* manner such that it classifies the states of a Markov chain. This issue was previously solved by using DNA computing techniques [2].

The structure of this work is as follows: in section 2 we introduce the basic concepts of information theory and membrane computing used in the following. In section 3, we propose a cell-like P system that models any given Markov source. In section 4, we provide some conclusions and describe our work in progress regarding this work.

2 Basic concepts.

In this section, we provide basic concepts from information theory [4, 12] and membrane computing [8, 9]. Basically, we define Markov sources and the main ingredients to analyze their behavior, and basic models for membrane computing such as cell-like P systems together with probabilistic methods to perform stochastics computations.

Definition 1. *An information source is defined by the pair* (A, P)*, where* A *is an alphabet and* $P : A \to [0, .., 1]$ *is a probability distribution over the set of symbols. We say that an information source is*

- **memoryless** *if the probabilities of every symbol that the source outputs at time* t*, let say* $p(a_t)$*, does not depend on the previous outputs of the source.*
- *a* **m-th order Markov source** *if the probabilities of every symbol that the source outputs at time* t*, depends on the previous* m *outputs of the source.*

It is well known that a m-th order Markov source can be represented in a diagrammatic way by graphs where the nodes summarizes the previous history of the source (up to m previous outputs), and the edges represent the probabilities for the next outputs.

Definition 2. *A stochastic P system with active membranes of degree* $m \geq 1$ *is defined by the tuple* $\Pi = (V, H, \mu, w_1, w_2, \cdots, w_m, R, i_0)$ *where*

1. V *is the alphabet of objects*
2. H *is the alphabet of labels for membranes*
3. μ *is the initial membrane structure, of degree* m*, with all membranes labeled with elements of* H*. Polarizations are not considered in the system. A membrane with label* h *is represented as* $[\ ]_h$
4. $w_1, w_2, \cdots, w_m$ *are strings over* V *specifying the multiset of objects initially in the regions defined by* μ
5. R *is a finite set of rules of the following types*
   (a) $[v \stackrel{p}{\to} w]_h$ *with* $v, w \in V^*$ *(evolution rules)*
   (b) $v[\ ]_h \stackrel{p}{\to} [w]_h$ *with* $v, w \in V^*$ *('in' communication rules)*
   (c) $[v]_h \stackrel{p}{\to} w[\ ]_h$ *with* $v, w \in V^*$ *('out' communication rules)*
   (d) $[v]_h \stackrel{p}{\to} [\ [w]_j]_h$ *with* $v, w \in V^*$ *(membrane creation with object evolution)*
   (e) $[\ [\ u\ ]_{h_1}\ ]_{h_2}\ [\ ]_{h_3} \stackrel{p}{\to} [\ ]_{h_2} [\ [\ w]_{h_1}\ ]_{h_3}$ *(membrane movement)*
6. $i_0 \in \{0, \cdots, m\} \cup \{\infty\}$ *indicates the region where the result of a computation is obtained (*∞ *represents the environment).*

The rules of the P system are applied in a maximally parallel manner according to a probability distribution. The computation of the system finishes whenever no rule can be applied. A configuration of the system at time t during a computation is defined by the membrane structure μ_t and the multisets of objects at every region in μ_t. Every rule $\alpha \stackrel{p}{\longrightarrow} \beta$, has a parameter $p \in [0, .., 1]$ that is a probability value that models the stochastic aspects necessary for a reliable simulation of a Markov source. Despite having only one environment modeled as a cell-like P-system, one approach to stochastic simulation can be similar to that developed for multi-environment P systems as in [10]. The mechanism for the application of the rules and the distribution of probabilities can be applied by following the the DCBA algorithm [6] (in this case we consider only one environment), or directly by applying Gillespie's algorithm [7].

3 A cell-like representation of a m-th order Markov source

In this section, we describe the way in which a P system can be constructed to define a m-th order Markov source with alphabet A. We will assume that the Markov source is defined by a graph $G = (V_G, E_G)$ where V_G is the set of states that summarize the previous m symbols emitted by the source, and E_G is a transition from one state to other with a probability as weight (hence, the edges define the next symbol probabilities).

From $G = (V_G, E_G)$, where V_G contains n different states or nodes, we can build a P system Π with the following ingredients:

- $V = \{o_1, o_2, \cdots, o_n\} \cup \{o\} \cup A$
- $H = \{0, 1, 2, \cdots, n\}$
- Initially, $w_0 = o_k$ for $k \in \{1, \cdots, n\}$ (the initial state), and $w_i = \lambda$ for $i \in \{1, \cdots, n\}$.
- The membrane structure μ is defined as $[[]_1[]_2 \cdots []_n]_0$.
- If the edge $i \xrightarrow{p} j \in E_G$, then we define the rule $[o]_i \xrightarrow{p} []_i o_j a$ where a is the suffix symbol defined by the state j
- for every symbol o_j, we define the rule $[o_j[]_j]_0 \to [[o]_j]_0$
- for every symbol $a \in A$, we define the rule $[a]_0 \to []_0 a$ (the symbol a is emitted to the environment as a part of a Markov chain).
- $i_0 = \infty$

Example 1. Let us consider the Markov source of first order with alphabet $\{a, b, c\}$ showed in figure 1. Then, we can build the P system showed in figure 2 with initial state a.

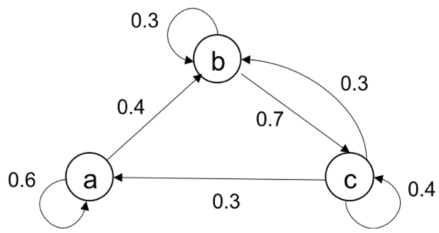

Fig. 1. A Markov source of first order

4 Conclusions and work in progress

We have proposed a method to build a cell-like P system for any given m-th order Markov source. The probabilities attached to the P systems rules reflect the transition probabilities in the Markov source. Then, the output of the P system is obtained in the environment by using algorithms for stochastic simulations such as Gillespie's, DCBA and others.

Our work in progress include the following aspects:

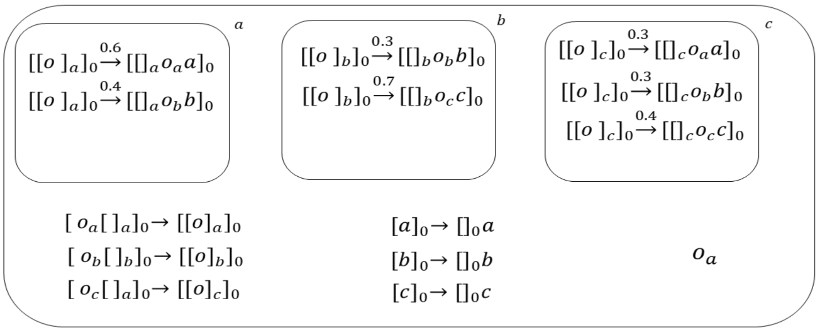

Fig. 2. A P system for the Markov source of Fig. 1 with an initial state 'a'.

1. The definition of a SN P system to simulate a m-th Markov source. In this case, the spikes train will encode the Markov chain.
2. The proposal of a semi-uniform solution to the definition of P systems for Markov sources. In this case, a P system with membrane creation rules defines the P system for the Markov source provided that an encoding of the source is given as input.

Observe that within this approach, we can introduce a Markov source in the framework of membrane computing by using only P systems. This could be frutiful to integrate stochastic information sources in simulations systems such as the one described in [1].

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